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SOFTWARE FOR THE KINEMATIC SYNTHESIS OF COUPLER-DRIVEN SPHERICAL FOUR-BAR MECHANISMS

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ABSTRACT

A spatial analogue of the Stephenson III six-bar mechanism can be formed by the connection of an SPS chain to the coupler of a spherical four-bar linkage. With the prismatic joint actuated, the spherical four-bar is driven via a force applied directly to the coupler. This linkage is termed the coupler-driven spherical four-bar mechanism, and defines an alternative to the typical scheme of actuating the four-bar via a torque applied at the input link. This paper presents software developed to assist in the kinematic synthesis of these mechanisms. In the first stage of the design, a circuit-defect free spherical four-bar is dimensioned with the capacity to guide a rigid body through two orientations. The second stage of the design is to locate the SPS leg such that the four-bar is smoothly drivable between the orientations.

INTRODUCTION

A planar Stephenson III six-bar is classically presented as a four-bar mechanism with an additional 3R chain connecting ground to coupler. The R represents a revolute joint. See Norton [1] and Erdman & Sandor [2] for details on this one degree-of-freedom linkage. The 3R chain provides either a second output from the mechanism or provides an alternate input for actuating the mechanism. In the latter case, the four-bar mechanism dictates the motion and the 3R chain provides the input to drive the mechanism over the needed range of motion. Used in this capacity, the 3R chain may be replaced with an RPR chain to introduce a prismatic joint for an alternate actuation scheme. The Stephenson III six-bar is readily reconsidered as a spherical device by requiring the seven R-joint axes intersect at a point (other than infinity). That is, the planar four-bar is replaced by a spherical four-bar, as in Fig. 1, and the planar 3R chain by a spherical 3R chain.

One of the implementation challenges associated with the spherical four-bar mechanism (s4R) is the generation of large internal loading. These significant loads arise from a combination of factors including the closed-chain architecture, the load being offset from its axis of rotation, and the driving of the system via an input torque at a fixed axis [3]. To alleviate some of these concerns, the Stephenson III is further reconsidered. In this work, an SPS leg is connnected directly to the coupler of an s4R. The S represents a spherical joint. This mechanism is termed the coupler-driven spherical four-bar (CDs4R) and is depicted in Fig. 2. With the s4R designed to produce the desired reorientation of a body, the SPS leg may then be sized with the goal of effectively moving the load. This paper presents a consideration of the kinematic synthesis of the SPS chain only, and excludes from consideration the statics of the device. Moreover, the focus herein is on two orientation synthesis for generating the spherical four-bar, where the synthesis procedure for the SPS chain may be applied to any s4R with a specified range of motion.

Low degree-of-freedom solutions to the two position spatial rigid-body guidance problem have been thoroughly considered. Two-jointed chains defined by a combination of spherical, revolute, cylindrical, helical, and prismatic joints are included in works by Tsai and Roth [4], Suh and Radcliffe [5], and Bodduluri *et al.* [6]. Well-known is that a combination of two two-jointed revolute chains yields an s4R, a single degree-of-freedom mech-

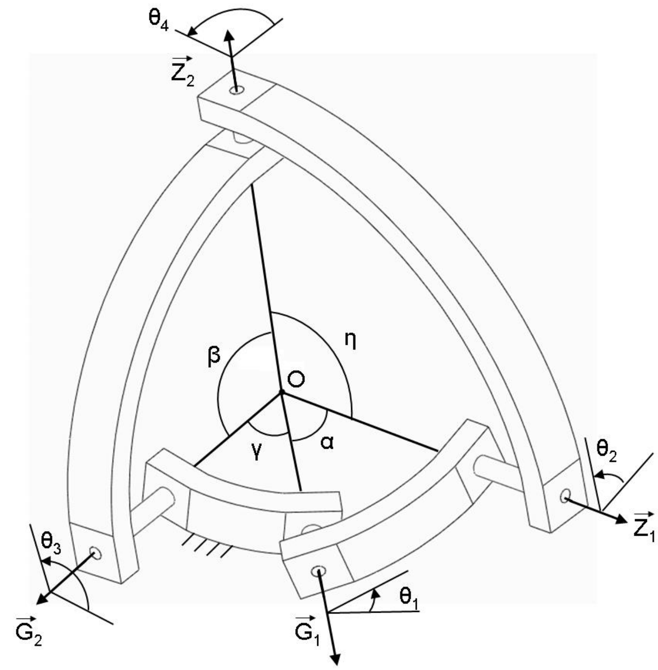

Figure 1. The motion of a spherical four-bar is defined by the link lengths α, β, η and γ.

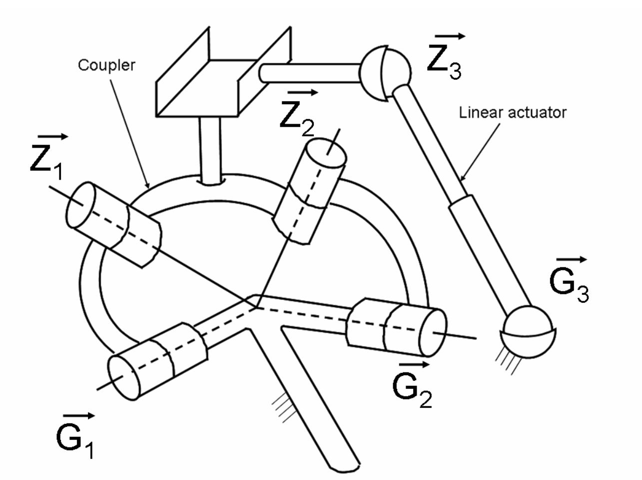

Figure 2. A CDs4R is composed of an s4R and an SPS driving leg.

anism capable of guiding a body through up to five orientations. The motion of the mechanism is determined by the angles α, β, γ, and η between the joint axes, where the angles are often referred to as the link *lengths*. Systematic methodologies for the determination of these angles for use of the s4R in rigid body guidance may be found in Chiang [7] and McCarthy [8].

As the number of orientations to be achieved by the spherical mechanism is increased, the set of usable solutions decreases. Part of this decrease is obviously due to the loss in design parameters resulting from the addition of constraints. Even though solutions are readily generated, however, the mechanism defects associated with circuits, branches and order [9] [10] can eliminate a large percentage of the solutions. Consider the case of four orientations, with a two-parameter set of s4R mechanisms to select from, where the order, branch and circuit defect free mechanism set can be extremely limited or even non-existent [11] [12]. An additional advantage of considering the SPS chain for actuation is that the branch points associated with driving the s4R from an input are eliminated, opening the solution space to a larger set of s4R mechanisms.

In order to synthesize CDs4Rs to achieve two orientation, interactive software in the Matlab^{TM} environment is proposed to facilitate navigation of the continuum of solutions. The software provides tools for navigating the six-parameter space of solutions to the s4R design problem, and then navigating the potential solutions for the SPS chain to realize a functioning CDs4R mechanism. Finally, the software was developed to have a user-friendly capacity.

TWO ORIENTATION SYNTHESIS

The specification of two orientations initiates the synthesis process. To utilize the intuition from familiarity with a globe, the orientations are modelled as being on the surface of a sphere rather than at the origin. The orientations are displaced outward along the x-axis. This allows orientation specification to occur by adding a roll angle to the familiar specification of longitude and latitude. Hence, orientation i, for $i = 1,2$, is defined by a longitude (λ_i), a latitude (ϕ_i) and a roll (ψ_i):

$$A_i = Z(\lambda_i)Y(-\phi_i)X(\psi_i) \tag{1}$$

where

$$Z(\lambda) = \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$Y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \tag{3}$$

$$X(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix} \tag{4}$$

Note that the negative sign in Eq. 1 arises due to the fact that a positive latitude should rotate the x-axis toward the north pole.

Continuing with notions from the globe, a latitude and longitude may be used to specify the directions of the axes of rotation of the revolute joints in the s4R. For the fixed axes $\vec{G}_i$, let m_i be the longitude and n_i be the latitude, then,

$$\vec{G}_i = \begin{Bmatrix} \cos(n_i)\cos(m_i) \\ \cos(n_i)\sin(m_i) \\ \sin(n_i) \end{Bmatrix}, i = 1,2. \tag{5}$$

Letting p_i be the longitude and q_i be the latitude of the moving axes $\vec{Z}_i$, relative to the fixed frame, when the mechanism is aligned with orientation 1 of the two orientations,

$$\vec{Z}_i = \begin{Bmatrix} \cos(q_i)\cos(p_i) \\ \cos(q_i)\sin(p_i) \\ \sin(q_i) \end{Bmatrix}, i = 1,2. \tag{6}$$

Note that the vectors $\vec{G}_i$ and $\vec{Z}_i$ are of unit length. Given that the vectors $\vec{Z}_i$ are known relative to the fixed frame when the mechanism is aligned at the first orientation,

$$\vec{Z}_{i2} = A_2 A_1^T \vec{Z}_i \tag{7}$$

yields the vectors $\vec{Z}_{i2}$, the moving axis directions relative to the fixed frame when the mechanism is aligned at the second orientation.

Observe that a rigid body fixing the angle between axes $\vec{G}_i$ and $\vec{Z}_i$ (at the first orientation) must hold the same angle between axes $\vec{G}_i$ and $\vec{Z}_{i2}$ (at the second orientation). Thus,

$$\vec{G}_i \cdot \vec{Z}_i = \vec{G}_i \cdot \vec{Z}_{i2}, \tag{8}$$

or,

$$\vec{G}_i^T (I - A_2 A_1^T)\vec{Z}_i = 0, \tag{9}$$

which is a single equation in the four unknown angles m_i, n_i, p_i, and q_i. Any three of these angles may be selected and the remaining angle is readily determined. In the software described below, the following solution methodology is integrated. Given the values for m_i, n_i, and p_i, the vector $\vec{G}_i$ is known. Eq. 9 simplifies to $u_1\cos(q_1) + u_2\sin(q_1)$ where u_1 and u_2 are known. In this way, m_1, n_1, p_1, m_2, n_2, and p_2 are selected, and q_1 and q_2 are solved for to produce the description of an s4R consistent with A_1 and A_2.

SOLUTION RECTIFICATION

The procedure in the previous section produces a mechanism that can be assembled in the two orientations. For so few orientations, the order defect is not a concern. The branch defect occurs when the mechanism must be driven through the singularity (or toggle) position. As this depends on the choice of actuated joint, and the intent is not to utilize an input at $\vec{G}_1$ or $\vec{G}_2$, assessing this defect relative to the current mechanism has little value. Finally, the circuit defect occurs when the mechanism must be disassembled to move from one orientation to the other. Thus, the only s4R mechanisms rejected at this stage of the synthesis methodology are those with a circuit defect between the orientations.

The s4R link lengths can be determined from the direction vectors associated with revolute joint axes,

$$\alpha = \arccos(\vec{G}_1 \cdot \vec{Z}_1), \tag{10}$$

$$\beta = \arccos(\vec{G}_2 \cdot \vec{Z}_2), \tag{11}$$

$$\gamma = \arccos(\vec{G}_1 \cdot \vec{G}_2), \text{ and} \tag{12}$$

$$\eta = \arccos(\vec{Z}_1 \cdot \vec{Z}_2). \tag{13}$$

A local reference frame is attached to the mechanism as follows. The x-axis of the frame is aligned with $\vec{G}_1$, the y-axis of the frame is in the plane of vectors $\vec{G}_1$ and $\vec{G}_2$, and the z-axis is in the direction of $\vec{G}_1 \times \vec{G}_2$. From this reference frame, and using the joint angles as defined in Fig. 1, the loop closure equation is seen to be

$$X(\theta_1)Z(\alpha)X(\theta_2)Z(\eta) = Z(\gamma)X(\theta_3)Z(\beta)X(\theta_4) \tag{14}$$

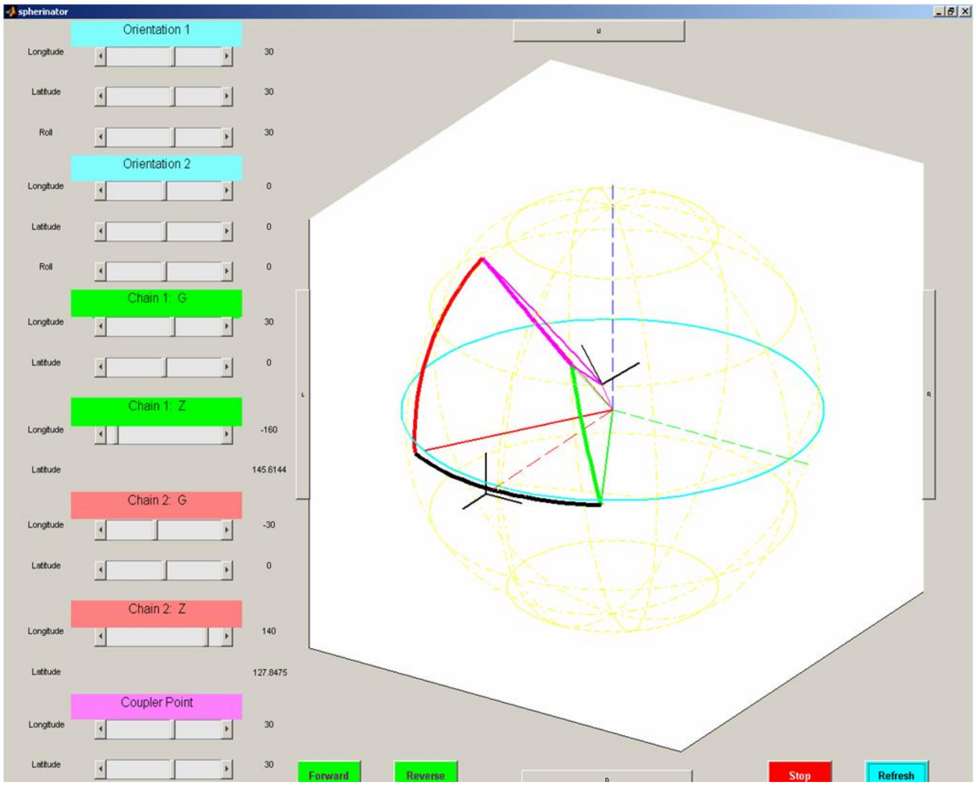

Figure 3. The Spherinator design environment includes input sliders and a display of the mechanism.

where the X and Z terms are rotation matrices defined by Eqs. 4 and 2 respectively. Given any joint angle in the mechanism, the remaining three are determined via Eq. 14.

Once the mechanism is synthesized, it is analyzed to inspect for the presence of the circuit defect. As only Grashof mechanisms are susceptible to this defect [13], the mechanism must first be classified [14]. Any non-Grashof mechanism is immediately acceptable. If the mechanism is Grashof, the further distinction of crank-rocker (α is shortest), rocker-crank (β is shortest), Grashof double-rocker (η is shortest) or drag-link (γ is shortest) is required. Note that the concept of shortest link is only applicable when the link lengths have been determined in such a way that no two sum to greater than 180 deg and only one is greater than 90 deg. Next, calculate the values of θ_{1i}, θ_{2i}, θ_{3i}, and θ_{4i}, the four joint values that satisfy Eq. 14 at orientation i. These angles should be determined such that they are in the range -180 deg $< \theta <= 180$ deg. If the mechanism is a crank-rocker or drag-link, it is free of the circuit defect if $\text{sign}(\theta_{41}) = \text{sign}(\theta_{42})$. If the mechanism is a Grashof double-rocker, it is free of the circuit defect if $\text{sign}(\theta_{31}) = \text{sign}(\theta_{32})$. If the mechanism is a rocker-crank, it is free of the circuit defect if $\text{sign}(\theta_{21}) = \text{sign}(\theta_{22})$. Note that these angle checks are not unique and that for each mechanism type a different joint angle in the mechanism could be checked. For example, a double-rocker is also free of the circuit defect if $\text{sign}(\theta_{21}) = \text{sign}(\theta_{22})$. If the mechanism suffers from a circuit defect, a different mechanism must be selected to proceed with the synthesis of a CDs4R.

SYNTHESIZING THE SPS CHAIN

Given an s4R free of the circuit defect, a driving SPS chain may now be synthesized. Because the focus of this work is determining potential S joint locations, the distance of the S joints from the center of the design sphere is set to one. Practically speaking, locating the S joints in this fashion increases the risk of collisions among mechanism parts and does not yield the best static loading characteristics for the overall mechanism. This analysis simply provides the allowable directions for the S joints, assuming they will be moved in or out from the center of the design sphere to avoid collisions and to improve the static loading situation.

The location of the fixed S joint, $\vec{G}_3$, is identified by the longitude and latitude angles m_3 and n_3, as in Eq. 5 for $i = 3$. The location of the S joint attached to the coupler, relative to the fixed frame, when the mechanism is aligned at the first position, is given by $\vec{Z}_3$. The vector $\vec{Z}_3$ is identified by the longitude and latitude angles p_3 and q_3, as in Eq. 6 for $i = 3$.

An effective combination of $\vec{G}_3$ and $\vec{Z}_3$ may be determined after generating the fixed axode of the coupler. At any instant, the coupler of the s4R has an axis of rotation that lies at the intersection of two planes. The first plane is defined by the rotation axes at the ends of the α link and the second plane is defined by the axes at the ends of the β link. Note that both planes include the point at the center of the design sphere. To find the axis of rotation of the coupler at an arbitrary location of θ_1, say $\theta_1 = \theta_{1i}$, the directions of the moving axes are calculated by solving for the remaining joint variables in Eq. 14, say $\theta_2 = \theta_{2i}$, $\theta_3 = \theta_{3i}$, and $\theta_4 = \theta_{4i}$. The location of the moving axis on the α link is

$$X(\theta_{1i})Z(\alpha)\begin{Bmatrix}1\\0\\0\end{Bmatrix}. \tag{15}$$

The location of the moving axis on the beta link is

$$Z(\gamma)X(\theta_{3i})Z(\beta)\begin{Bmatrix}1\\0\\0\end{Bmatrix}. \tag{16}$$

In this way, any direction on the axode may be generated.

Given that the SPS chain can only apply a force along the line of action between the S joints, the synthesis requirement is that this line cannot intersect the axis of rotation of the coupler over the range of motion. The configuration at which they intersect is the branch point for the CDs4R mechanism. Note that, as the s4R moves, the location of the instantaneous axis of rotation and the location of the S joint attached to the coupler change. At each location along the motion, a plane is eliminated from the possible locations for $\vec{G}_3$. Over the entire motion, an unusable volume is swept out. The selection of $\vec{G}_3$ outside this swept volume yields a CDs4R where the SPS chain has the capacity to drive the device between the two orientations in a singularity-free fashion.

SPHERINATOR

Spherinator is the title given to the software designed to implement the above theory in a user-friendly fashion. The complete Spherinator environment is shown in Fig. 3. Most of the window is occupied by the design sphere. Two orientations can be moved around on the sphere to intialize a design problem. The orientations are moved via slider-bars seen along the left-hand side. The six topmost slider-bars control the latitude, longitude, and roll angles for each position in the range of -180 to 180 degrees.

With two orientations specified, solutions can be explored using the next six slider-bars by selecting the latitude and longitude of the fixed axes, or $\vec{G}$, vectors and the longitudes of the moving axes, or $\vec{Z}$, vectors. The latitudes of $\vec{Z}$ are determined through use of Eq. 9. In this way, the s4R mechanism is always consistent with the two orientations shown. In fact, changes in the original orientations intitiate the calculation of these latitudes. As such, the two orientations and the s4R depicted are always consistent.

Other features include the color-coding of the links of the s4R. The α link is shown in green, the β link in red, the η link in pink, and the γ link in black. A warning window indicates if the current s4R suffers from the circuit-defect. Finally, the viewing angle of the design sphere may be chosen by the user.

Once the above tools have generated a candidate s4R, an SPS chain may be synthesized to complete the CDs4R. The two bottom-most slider-bars control $\vec{Z}_3$, the location of the moving S joint when the device is in the first orientation. Observe that the circuit-defect free s4R can traverse the distance between the two orientations along two separate paths. Pressing either the *Forward* or *Reverse* buttons located along the bottom of the window drives the linkage between the orientations. See Fig. 4. Selecting the *Forward* option increments θ_1 positively to start the motion, and the *Reverse* button increments the angle negatively. Both directions are needed to view the entire range of motion of the s4R mechanism on this circuit.

With $\vec{Z}_3$ and a path selected, valid locations for $\vec{G}_3$ are generated. The mechanism is driven over the path noting the plane of unusable $\vec{G}_3$ locations at each iteration. The intersection of each of these planes with the design sphere defines a great circle. Each great circle is indicated, as shown in Fig. 5, on a Mercator Projection-like plot. The regions swept by these great circles indicate the directions for the fixed S joint that, if chosen, will result in a branch defect between the initial and final orientations. Selecting outside the swept region results in a usable choice of $\vec{G}_3$. Combinations of positions, mechanisms, paths, and moving S joint locations may be selected that result in no possible locations for the fixed S joint to actuate the CDs4R mechanism. If valid locations for $\vec{G}_3$ exist, selecting its location in the unswept region of the plot finishes the CDs4R. A final plot is generated indicating the length of the SPS chain over the course of the motion. A viable choice of $\vec{G}_3$ yields a monotonic motion of the SPS chain as in Fig. 6.

CONCLUSION

A design algorithm and software for the kinematic synthesis of the coupler-driven spherical four-bar (CDs4R) mechanism were presented. The CDs4R is defined by an SPS chain connecting from the ground frame directly to the coupler, with the actuation provided via the P joint. This mechanism is being investigated as an alternate to the more classical notion of driving a spherical four-bar via a torque applied at one of the fixed axes. A significant potential advantage of this architecture is the lower internal forces developed in the mechanism during operation.

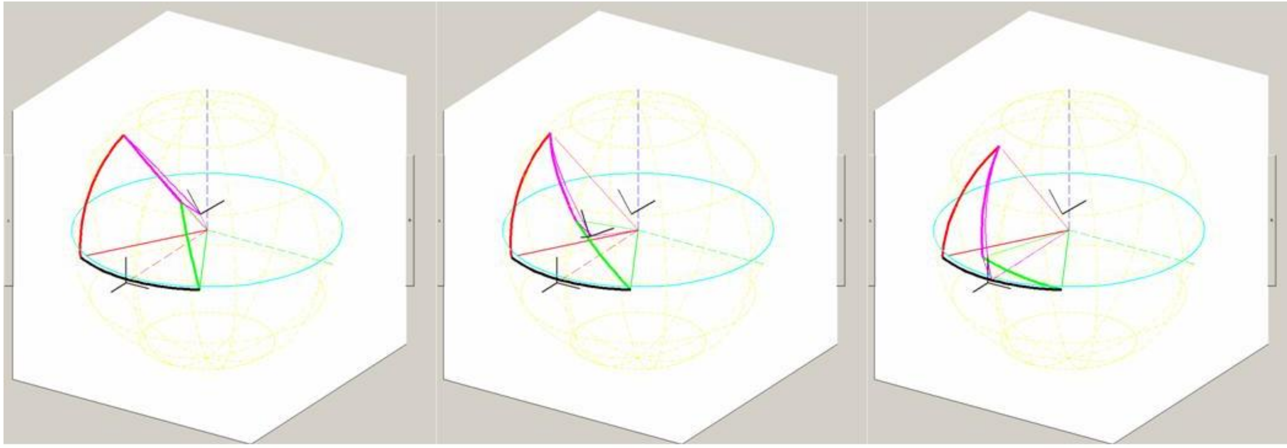

Figure 4. Activating the Forward or Reverse buttons drives the s4R mechanism.

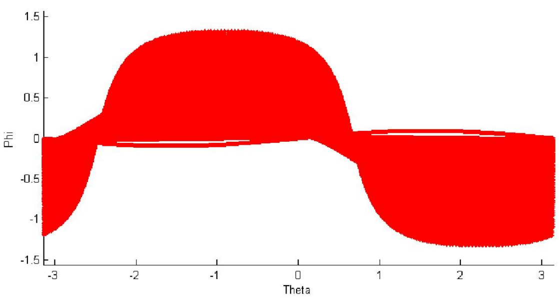

Figure 5. Locating the fixed S joint in the white (unswept) region produces a functional CDs4R.

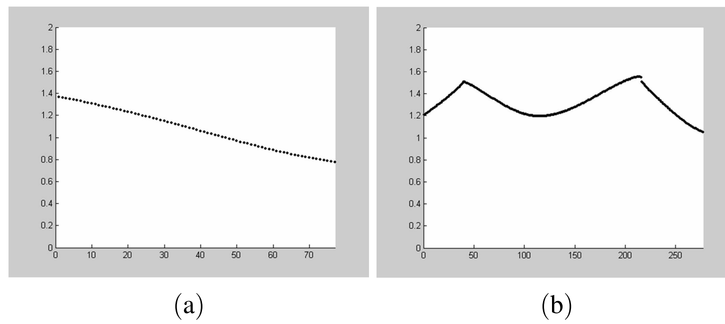

Figure 6. The final plot is monotonic for an acceptable choice of fixed S joint location (a), and non-monotonic for an unacceptable choice (b).

The synthesis methodology revealed that, for two orientations, a mechanism free of circuit defects is a potentially usable solution. Morever, the procedure for designing the final SPS chain could be applied to any s4R mechanism with a known range of motion.

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